

Part A

- 1. C
- 2. D
- 3. B
- 4. B
- 5. D
- 6. D
- 7. C
- 8. B
- 9. A
- 10. D
- 11. C
- 12. C
- 13. A
- 14. B
- 15. D
- 16. B
- 17. D
- 18. C
- 19. C
- 20. C
- 21. B

- 22. C
- 23. C
- 24. A
- 25. B
- 26. C
- 27. C
- 28. D
- 29. D
- 30. D

Part B

- 31. A
- 32. C
- 33. A
- 34. B
- 35. B
- 36. B
- 37. B
- 38. C
- 39. B
- 40. C
- 41. B
- 42. C
- 43. B
- 44. A
- 45. D



CALCULUS AB

SECTION I, Part A Solutions

1. Correct answer: (C)

To find the integral, we need to use integration by parts. Let $u = e^{2x}$ and $du = 2e^{2x} dx$ with $dv = \cos x dx$ and $v = \sin x$.

$$\int e^{2x} \cos x dx = e^{2x} \sin x - 2 \int \sin x e^{2x} dx$$

Now we need to use the integration by parts one more time to find

$\int e^{2x} \sin x dx$. Let $u = e^{2x}$ and $du = 2e^{2x} dx$ with $dv = \sin x dx$ and $v = -\cos x$.

$$\int e^{2x} \sin x dx = -e^{2x} \cos x + 2 \int e^{2x} \cos x dx$$

$$\int e^{2x} \cos x dx = e^{2x} \sin x - 2 \left(-e^{2x} \cos x + 2 \int e^{2x} \cos x dx \right)$$

$$\int e^{2x} \cos x dx = e^{2x} \sin x + 2e^{2x} \cos x - 4 \int e^{2x} \cos x dx$$

$$5 \int e^{2x} \cos x dx = e^{2x} \sin x + 2e^{2x} \cos x$$

$$\int e^{2x} \cos x dx = \frac{e^{2x}(\sin x + 2 \cos x)}{5} + C$$

2. Correct answer: (D)

To find $f'(x)$, we need to use the chain rule.

$$\begin{aligned}\frac{d}{dx}(\cos(e^{-\tan x})) &= -\sin(e^{-\tan x}) \cdot \frac{d}{dx}(e^{-\tan x}) \\ &= -\sin(e^{-\tan x}) \cdot (-\sec^2 x e^{-\tan x}) \\ &= e^{-\tan x} \sin(e^{-\tan x}) \sec^2 x\end{aligned}$$

3. Correct answer: (B)

We don't need to use the Fundamental Theorem here, we only need to substitute $x = 2$ into the integral.

$$F(2) = \int_0^2 t^3 + 2 \, dt = \left. \frac{t^4}{4} + 2t \right|_0^2 = \left(\frac{16}{4} + 4 \right) = 8$$

4. Correct answer: (B)

First, find $f'(x)$.

$$\begin{aligned}f'(x) &= 2 \cdot 2 \cos x \cdot \frac{d}{dx}(\cos x) + \sec^2 x \\ &= 4 \cos x(-\sin x) + \sec^2 x \\ &= -4 \sin x \cos x + \sec^2 x\end{aligned}$$

Now find $f'\left(\frac{\pi}{6}\right)$.

$$f'\left(\frac{\pi}{6}\right) = -4 \sin \frac{\pi}{6} \cos \frac{\pi}{6} + \sec^2 \frac{\pi}{6}$$

$$= -4 \left(\frac{1}{2} \right) \left(\frac{\sqrt{3}}{2} \right) + \left(\frac{2\sqrt{3}}{3} \right)^2$$

$$= -\sqrt{3} + \frac{4}{3}$$

5. Correct answer: (D)

If the function is increasing and concave up, then the first and the second derivatives will be positive.

$$f'(x) = 5x^4 - 30x^2$$

Then we need to find where the function is positive.

$$5x^4 - 30x^2 = 0$$

$$5x^2(x^2 - 6) = 0$$

$$x = -\sqrt{6}, 0, \sqrt{6}$$

We need to evaluate f' at test values $x = -3$, $x = -1$, $x = 1$, and $x = 3$.

$$f'(-3) = 5(-3)^4 - 30(-3)^2 = 405 - 270 = 135 > 0$$

$$f'(-1) = 5(-1)^4 - 30(-1)^2 = 5 - 30 = -25 < 0$$

$$f'(1) = 5(1)^4 - 30(1)^2 = 5 - 30 = -25 < 0$$

$$f'(3) = 5(3)^4 - 30(3)^2 = 405 - 270 = 135 > 0$$

Therefore, the function increases on the interval $(-\infty, -\sqrt{6})$ and $(\sqrt{6}, \infty)$.

$$f''(x) = 20x^3 - 60x$$

$$20x^3 - 60x = 0$$

$$20x(x^2 - 3) = 0$$

$$x = -\sqrt{3}, 0, \sqrt{3}$$

We need to evaluate f'' at the test values of $x = -2$, $x = -1$, $x = 1$, and $x = 2$.

$$f''(-2) = 20(-2)^3 - 60(-2) = -160 + 120 = -40 < 0$$

$$f''(-1) = 20(-1)^3 - 60(-1) = -20 + 60 = 40 > 0$$

$$f''(1) = 20(1)^3 - 60(1) = 20 - 60 = -40 < 0$$

$$f''(2) = 20(2)^3 - 60(2) = 160 - 120 = 40 > 0$$

Therefore, the function is concave up on the intervals $(-\sqrt{3}, 0)$ and $(\sqrt{3}, \infty)$.

Therefore, the function is increasing and concave up on the interval $(\sqrt{6}, \infty)$, because $\sqrt{3} < \sqrt{6}$.

6. Correct answer: (D)

We should take the derivative of each of I, II, and III, and see what we get.

$$\frac{d}{dx} \left(\frac{\sin^3 x}{3} + \sin^2 x \right) = \frac{3 \sin^2 x \cos x}{3} + 2 \sin x \cos x = \sin^2 x \cos x + 2 \sin x \cos x$$

$$\frac{d}{dx} \left(\frac{2 + \sin^3 x}{3} - \cos^2 x \right) = \frac{3 \sin^2 x \cos x}{3} - 2 \cos x (-\sin x) = \sin^2 x \cos x + 2 \cos x \sin x$$

$$\frac{d}{dx} \left(\frac{\cos^3 x}{3} - \frac{\cos^2 x}{2} \right) = \frac{3 \cos^2 x (-\sin x)}{3} - \frac{2 \cos x (-\sin x)}{2} = -\sin x \cos^2 x + \cos x \sin x$$

Therefore, I and II are both antiderivatives of $f(x)$.

7. Correct answer: (C)

$$\frac{dy}{dt} = 2 \sec^2 2t$$

$$\frac{dx}{dt} = 2 \cos 2t$$

Then

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2 \sec^2 2t}{2 \cos 2t} = \frac{\sec^2 2t}{\cos 2t} = \frac{\frac{1}{\cos^2 2t}}{\cos 2t} = \frac{1}{\cos^3 2t}$$

8. Correct answer: (B)

We know that

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

Then

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n (2x)^n}{n!} = e^{-2x}$$

So

$$f(5) = e^{-2 \cdot 5} = e^{-10} = \frac{1}{e^{10}}$$

9. Correct answer: (A)

To find the radius of convergence, we need to use the ratio test.

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{(n+1)(x-5)^{2(n+1)}}{2^{n+1}} \cdot \frac{2^n}{n \cdot (x-5)^{2n}} \right|$$

$$= \lim_{n \rightarrow \infty} \left| \frac{(n+1)(x-5)^2}{2n} \right|$$

$$= \left| \frac{(x-5)^2}{2} \right| \lim_{n \rightarrow \infty} \left| \frac{n+1}{n} \right|$$

$$= \left| \frac{(x-5)^2}{2} \right|$$

$$\left| \frac{(x-5)^2}{2} \right| < 1$$

$$|(x-5)^2| < 2$$

$$|x-5| < \sqrt{2}$$

Therefore, the radius of convergence is $R = \sqrt{2}$.

10. Correct answer: (D)

To find the slope of the line tangent to the graph, we need to find $\frac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

We know that

$$x = r \cos \theta$$

$$y = r \sin \theta$$

Then

$$\frac{dx}{d\theta} = \frac{dr}{d\theta} \cos \theta - r \sin \theta$$

$$\frac{dy}{d\theta} = \frac{dr}{d\theta} \sin \theta + r \cos \theta$$

We get

$$\frac{dy}{dx} = \frac{\frac{dr}{d\theta} \sin \theta + r \cos \theta}{\frac{dr}{d\theta} \cos \theta - r \sin \theta}$$

Since $r = 4 + 5 \cos \theta$, then

$$\frac{dr}{d\theta} = -5 \sin \theta$$

$$\frac{dr}{d\theta} \left(\frac{\pi}{2} \right) = -5 \sin \frac{\pi}{2} = -5 \cdot 1 = -5$$

$$r\left(\frac{\pi}{2}\right) = 4 + 5 \cos \frac{\pi}{2} = 4 + 0 = 4$$

$$\frac{dy}{dx}\left(\frac{\pi}{2}\right) = \frac{-5 \sin \frac{\pi}{2} + 4 \cos \frac{\pi}{2}}{-5 \cos \frac{\pi}{2} - 4 \sin \frac{\pi}{2}} = \frac{-5 \cdot 1 + 4 \cdot 0}{-5 \cdot 0 - 4 \cdot 1} = \frac{-5}{-4} = \frac{5}{4}$$

11. Correct answer: (C)

Since f' is positive on $(-\infty, -2)$, f must be increasing over this same interval. Since f' is negative on $(-2, 0)$ and $(2, \infty)$, f must be decreasing over these same intervals.

12. Correct answer: (C)

$$f'(x) = 4x^3 - 30x^2 + 50x = 2x(x - 5)(2x - 5)$$

Use test values $x = -1$, $x = 1$, $x = 3$, $x = 6$.

$$f'(-1) = 4(-1)^3 - 30(-1)^2 + 50(-1) = -4 - 30 - 50 = -84 < 0$$

$$f'(1) = 4(1)^3 - 30(1)^2 + 50(1) = 4 - 30 + 50 = 24 > 0$$

Therefore, at $x = 0$ the function f has relative minimum because f' changes from negative to positive, and the only false statement is C.

13. Correct answer: (A)

f is decreasing when $f' < 0$. Since the numerator is non-negative, this is only when the denominator is negative.

$$x - 4 < 0$$

$$x < 4, \text{ or } (-\infty, 4)$$

14. Correct answer: (B)

Use the chain rule

$$h'(x) = g'(f'(x)) \cdot f''(x)$$

g'' will always be negative. Since $h'(2) = -3$, $f''(2)$ must be positive. Therefore, f must be concave up at $x = 2$.

15. Correct answer: (D)

The curve represented by the parametric equation has a horizontal tangent if $\frac{dy}{dt} = 0$ and $\frac{dx}{dt} \neq 0$.

First, find the derivatives of the parametric equations.

$$\frac{dx}{dt} = 3t^2 - 6t + 3 = 3(t^2 - 2t + 1) = 3(t - 1)^2$$

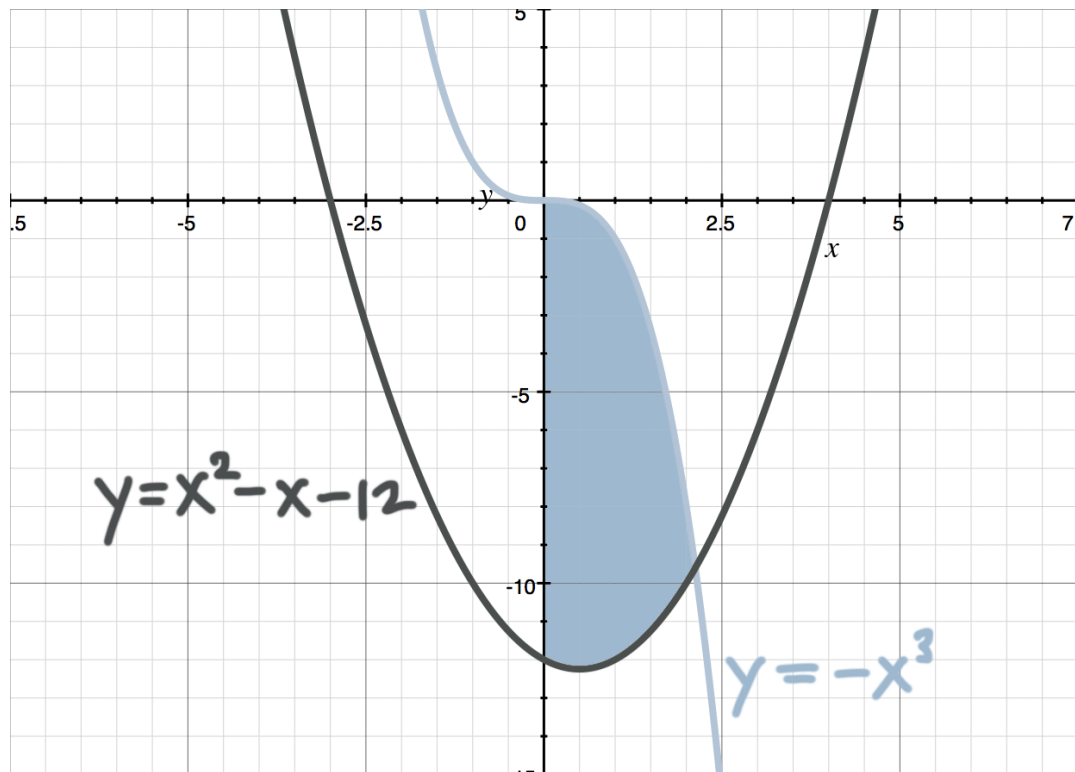
$$\frac{dy}{dt} = 2t - 4$$

$$\frac{dy}{dt} = 0 \text{ where } 2t - 4 = 0 \text{ and } t = 2$$

Therefore, the horizontal tangent occurs at $t = 2$.

16. Correct answer: (B)

Sketch the region.



The curve $y = -x^3$ is above $y = x^2 - x - 12$ over the entire interval.
Therefore,

$$\begin{aligned} A &= \int_0^2 -x^3 - (x^2 - x - 12) \, dx = \int_0^2 -x^3 - x^2 + x + 12 \, dx \\ &= -\frac{x^4}{4} - \frac{x^3}{3} + \frac{x^2}{2} + 12x \Big|_0^2 = -4 - \frac{8}{3} + 2 + 24 = \frac{58}{3} \end{aligned}$$

17. Correct answer: (D)

Remember that everything is differentiated with respect to x .

$$6x + 3y^2 \frac{dy}{dx} = 3y + 3x \frac{dy}{dx}$$

$$(3y^2 - 3x)\frac{dy}{dx} = 3y - 6x$$

$$\frac{dy}{dx} = \frac{3y - 6x}{3y^2 - 3x}$$

$$\frac{dy}{dx} = \frac{y - 2x}{y^2 - x}$$

18. Correct answer: (C)

We know that

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots + \frac{(-1)^n(x^{2n})}{(2n)!} + \dots$$

But if we multiply it by x , we get

$$\begin{aligned} x \cos x &= x \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots + \frac{(-1)^n(x^{2n})}{(2n)!} + \dots \right) \\ &= x - \frac{x^3}{2!} + \frac{x^5}{4!} - \frac{x^7}{6!} + \dots + \frac{(-1)^n x^{2n+1}}{(2n)!} + \dots \end{aligned}$$

19. Correct answer: (C)

We can rewrite the series as

$$1 + \frac{2^x}{2^2} + \frac{3^x}{3^2} + \frac{4^x}{4^2} + \dots + \frac{n^x}{n^2} + \dots = 1 + 2^{x-2} + 3^{x-2} + 4^{x-2} + \dots + n^{x-2} + \dots$$

$$= \sum_{n=1}^{\infty} \frac{1}{n^p} \text{ where } p = -(x-2) = 2-x$$

This is a p -series and it is convergent if $p > 1$. Therefore, the series is convergent if

$$2 - x > 1$$

$$-x > -1$$

$$x < 1$$

20. Correct answer: (C)

To find the average value of the function, we should apply the Mean Value Theorem.

$$\begin{aligned} f_{ave} &= \frac{1}{b-a} \int_a^b f(x) dx = \frac{1}{e-1} \int_1^e \frac{3}{x} dx = \frac{3}{e-1} \int_1^e \frac{1}{x} dx \\ &= \frac{3}{e-1} (\ln x) \Big|_1^e = \frac{3}{e-1} (\ln e - \ln 1) = \frac{3}{e-1} \end{aligned}$$

21. Correct answer: (B)

Note that limits at infinity resemble horizontal asymptotes. The limit does not exist when the degree of the numerator is greater than the degree of the denominator, and the limit is zero when the degree of the denominator is greater than the degree of the numerator. When the degrees are equal,

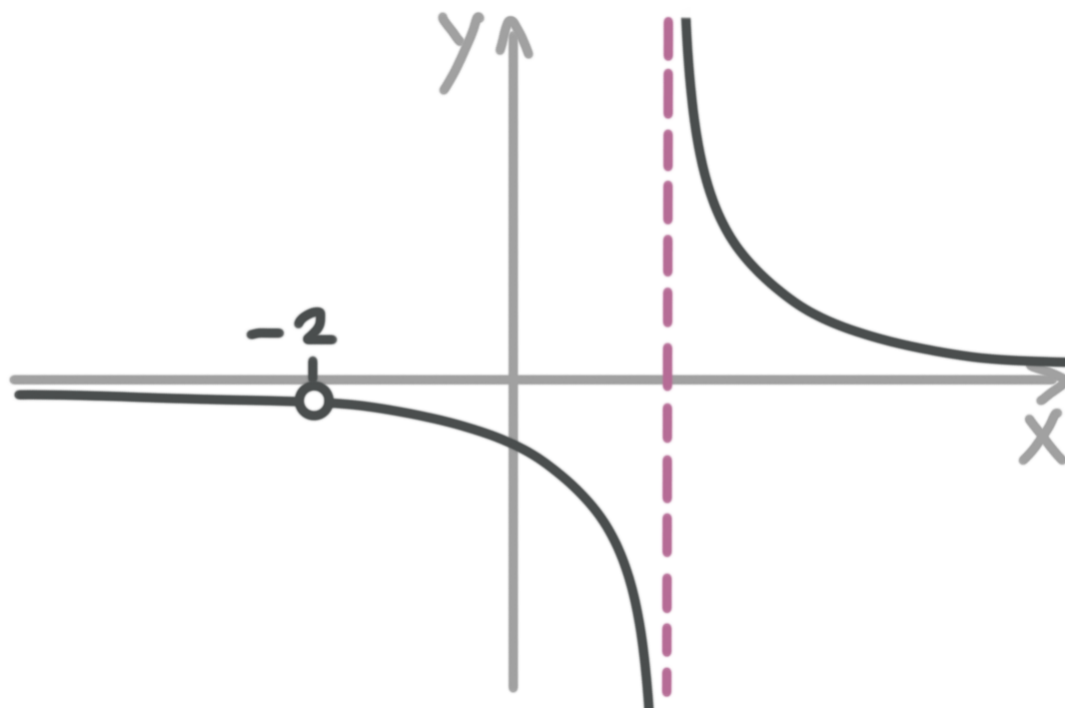
the limit is the ratio of the leading coefficients of the numerator and the denominator. Therefore, the limit is zero.

22. Correct answer: (C)

We can rewrite $f(x)$ as

$$f(x) = \frac{2 + x}{(x - 2)(x + 2)} = \frac{1}{x - 2}$$

with a point discontinuity at $x = -2$. Sketch a graph.



We can see that the function is concave up to the right of the asymptote, $(2, \infty)$.

23. Correct answer: (C)

$y = 4\sqrt{x}$ and $y = 4x^2$ have two intersection points, (0,0) and (1,4).

Since $\sqrt{x} > x^2$, then

$$A = \int_0^1 4\sqrt{x} - 4x^2 \, dx$$

24. Correct answer: (A)

This is just the derivative of $f(x) = 3 \cos 2x$ evaluated at $x = \frac{\pi}{4}$, $f'(x) = -6 \sin 2x$.

$$f' \left(\frac{\pi}{4} \right) = -6 \sin \left(2 \cdot \frac{\pi}{4} \right) = -6 \sin \frac{\pi}{2} = -6$$

25. Correct answer: (B)

Let $u = \sin x$ and $du = \cos x$. So we can rewrite the integral as

$$\int \sin^3 x \cos x \, dx = \int u^3 \, du = \frac{u^4}{4}$$

$$\frac{\sin^4 x}{4} \Big|_0^{\frac{\pi}{2}} = \frac{\left(\sin \frac{\pi}{2} \right)^4}{4} - \frac{(\sin 0)^4}{4} = \frac{1}{4} - 0 = \frac{1}{4}$$

26. Correct answer: (C)

For a particle to be at rest, both $\frac{dy}{dt}$ and $\frac{dx}{dt}$ must be zero.

$$\frac{dy}{dt} = 6 - 6t = 6(1 - t)$$

$$\frac{dy}{dt} = 0 \text{ when } t = 1$$

$$\frac{dx}{dt} = 6t^2 - 6t = 6t(t - 1)$$

$$\frac{dx}{dt} = 0 \text{ when } t = 0 \text{ and } t = 1$$

$\frac{dy}{dt}$ and $\frac{dx}{dt}$ both are zero at $t = 1$.

$$x(1) = 2 \cdot 1^3 - 3 \cdot 1^2 = 2 - 3 = -1$$

$$y(1) = 6 \cdot 1 - 3 \cdot 1^2 = 6 - 3 = 3$$

27. Correct answer: (C)

The one-sided limits are

$$\lim_{x \rightarrow 2^-} f(x) = 4 \ln 4$$

$$\lim_{x \rightarrow 2^+} f(x) = 4 \ln 4$$

Since $\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x)$, then $\lim_{x \rightarrow 2} f(x) = 4 \ln 4$.

28. Correct answer: (D)

$$\frac{x}{15} = \cos \theta \text{ so } x = 15 \cos \theta.$$

Differentiating with respect to t gives

$$\frac{dx}{dt} = -15 \sin \theta \frac{d\theta}{dt}$$

When $x = 9$ units, use the Pythagorean theorem to find the third side and then $\sin \theta$.

$$9^2 + h^2 = 15^2$$

$$h^2 = 225 - 81 = 144$$

$$h = \pm 12$$

Since the side length cannot be negative, then $h = 12$.

$$\sin \theta = \frac{h}{15} = \frac{12}{15}$$

$$\sin \theta = \frac{4}{5}$$

Since $\frac{d\theta}{dt} = -5$, we get

$$\frac{dx}{dt} = -15 \left(\frac{4}{5} \right) (-5) = 12 \cdot 5 = 60$$

x increases at a rate of 60 radians per minute.

29. Correct answer: (D)

We need to use quotient and chain rules to evaluate the derivative.

$$f'(x) = \frac{(-3x^2 \sin x^3)e^{2x^2} - 4xe^{2x^2} \cdot \cos x^3}{(e^{2x^2})^2}$$

$$= \frac{e^{2x^2}(-3x^2 \sin x^3 - 4x \cos x^3)}{(e^{2x^2})^2}$$

$$= \frac{-3x^2 \sin x^3 - 4x \cos x^3}{e^{2x^2}}$$

30. Correct answer: (D)

The function is increasing from $x = 0$ to $x = 1$ by $\frac{3}{2}$ squared units, which was determined by finding the area of the triangle. Then the function is increasing from $x = 1$ to $x = 3$ by 2 square units. From $x = 3$ to $x = 6$, it is increasing by $2 + 1 = 3$ square units. From $x = 6$ to $x = 8$ the function is decreasing by 2 square units. We can see that the function's value is greatest at $x = 6$ followed by at $x = 8$, then $x = 3$ and $x = 1$.

Therefore, $f(1) < f(3) < f(8) < f(6)$.

CALCULUS AB

SECTION I, Part B Solutions

31. Correct answer: (A)

First, solve for I. First, factor the largest power of n from both the numerator and denominator, and then take the limit.

$$\lim_{n \rightarrow \infty} \frac{2n}{3 - 4n} = \lim_{n \rightarrow \infty} \frac{2}{\frac{3}{n} - 4} = -\frac{2}{4} = -\frac{1}{2}$$

So the sequence converges.

Next, solve for II.

$$\lim_{n \rightarrow \infty} \frac{e^{3n}}{2 - e^{2n}} = \lim_{n \rightarrow \infty} \frac{3e^{3n}}{-2e^{2n}} = \lim_{n \rightarrow \infty} -\frac{3}{2}e^n = -\infty$$

So the sequence diverges.

Finally, solve for III.

$$\lim_{n \rightarrow \infty} \frac{e^{3n}}{3n} = \lim_{n \rightarrow \infty} \frac{3e^{3n}}{3} = \lim_{n \rightarrow \infty} e^{3n} = \infty$$

So the sequence diverges.

32. Correct answer: (C)

Use partial fractions.

$$\frac{1}{(x-4)(x-2)} = \frac{A}{x-4} + \frac{B}{x-2}$$

$$A(x-2) + B(x-4) = 1$$

If $x = 2$,

$$0 + B(-2) = 1$$

$$B = -\frac{1}{2}$$

If $x = 4$,

$$2A + 0 = 1$$

$$A = \frac{1}{2}$$

Therefore

$$\frac{1}{(x-4)(x-2)} = \frac{1}{2} \left(\frac{1}{x-4} - \frac{1}{x-2} \right)$$

$$\int \frac{1}{(x-4)(x-2)} dx = \frac{1}{2} \int \frac{1}{x-4} dx - \frac{1}{2} \int \frac{1}{x-2} dx$$

$$= \frac{1}{2} \ln|x-4| - \frac{1}{2} \ln|x-2| + C$$

$$= \frac{1}{2} \ln \left| \frac{x-4}{x-2} \right| + C$$

33. Correct answer: (A)

The graph of $y = x^2 + 2x - 8$ is a parabola that changes from positive to negative at $x = -4$ and from negative to positive at $x = 2$. Since g is always positive, f' does not change sign.

Therefore, f has a relative maximum at $x = -4$ and a relative minimum at $x = 2$.

34. Correct answer: (B)

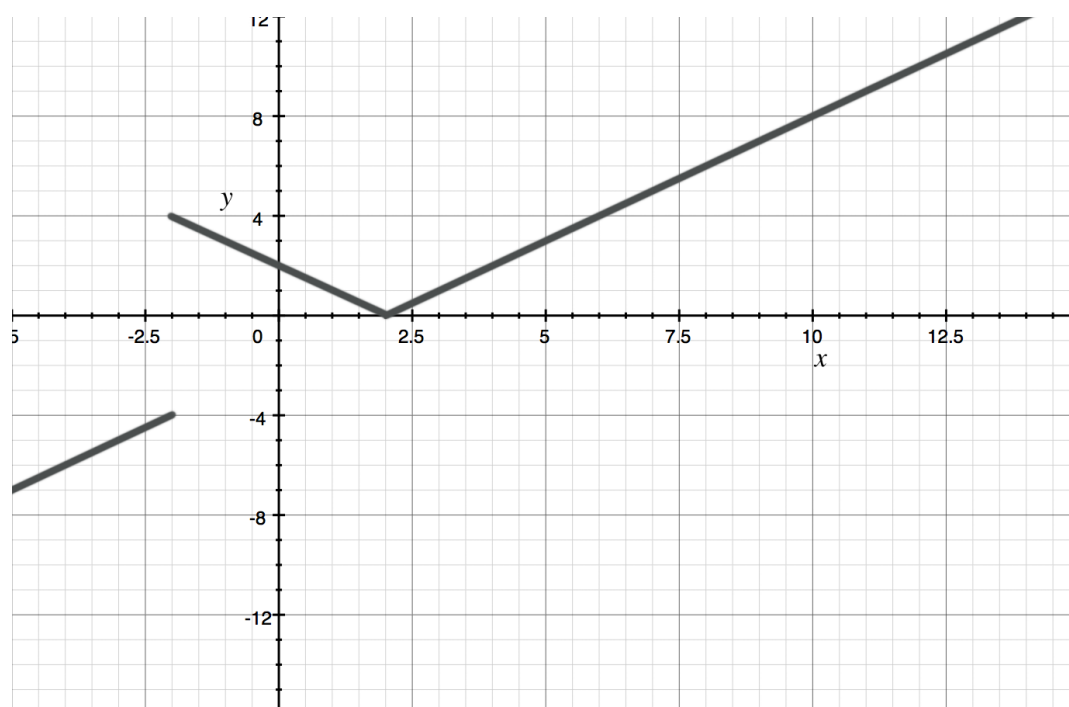
The area of a circle is $A = \pi r^2$. Differentiating with respect to t gives us

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

We know that $C = 2\pi r$ and $\frac{dr}{dt} = 0.15$, so $\frac{dA}{dt} = 0.15C$.

35. Correct answer: (B)

Graph the function.



We see that it is continuous at $x = 2$ and has a jump discontinuity at $x = -2$. Therefore, it is not differentiable at $x = -2$. Since the graph is decreasing before and increasing after $x = 2$, there is a local minimum at $x = 2$. Therefore, I and II are false.

36. Correct answer: (B)

Since the lines are parallel, then the slopes are equal.

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{y'(t)}{x'(t)}$$

$$\frac{dy}{dx} = 2 = \frac{y'(t)}{x'(t)} = \frac{2ke^{kt}}{\frac{2}{t+3}}$$

So

$$2 = ke^{kt}(t+3)$$

Substitute $t = 3$ and solve for k .

$$2 = ke^{3t} \cdot 6$$

$$k \approx 0.189$$

37. Correct answer: (B)

Area is equal to the area of circle minus the area of the rose. The area of the circle is $\pi r^2 = 3^2\pi = 9\pi$.

Find the area of the rose. $r = 0$ when $3 \sin 2\theta = 0$, so

$$2\theta = 0, \pi$$

$$\theta = 0, \frac{\pi}{2}$$

Therefore, the values $\theta = 0$ to $\theta = \frac{\pi}{2}$ trace out the first petal of the rose, so the area of the first petal is

$$\begin{aligned} A &= \frac{1}{2} \int_0^{\frac{\pi}{2}} (3 \sin 2\theta)^2 d\theta = \frac{1}{2} \int_0^{\frac{\pi}{2}} 9 \sin^2 2\theta d\theta = \frac{9}{2} \int_0^{\frac{\pi}{2}} \sin^2 2\theta d\theta \\ &= \frac{9}{2} \int_0^{\frac{\pi}{2}} \frac{1 - \cos 4\theta}{2} d\theta = \frac{9}{4} \int_0^{\frac{\pi}{2}} 1 - \cos 4\theta d\theta = \frac{9\pi}{8} \end{aligned}$$

All petals have the same area. Therefore, the area of the rose is

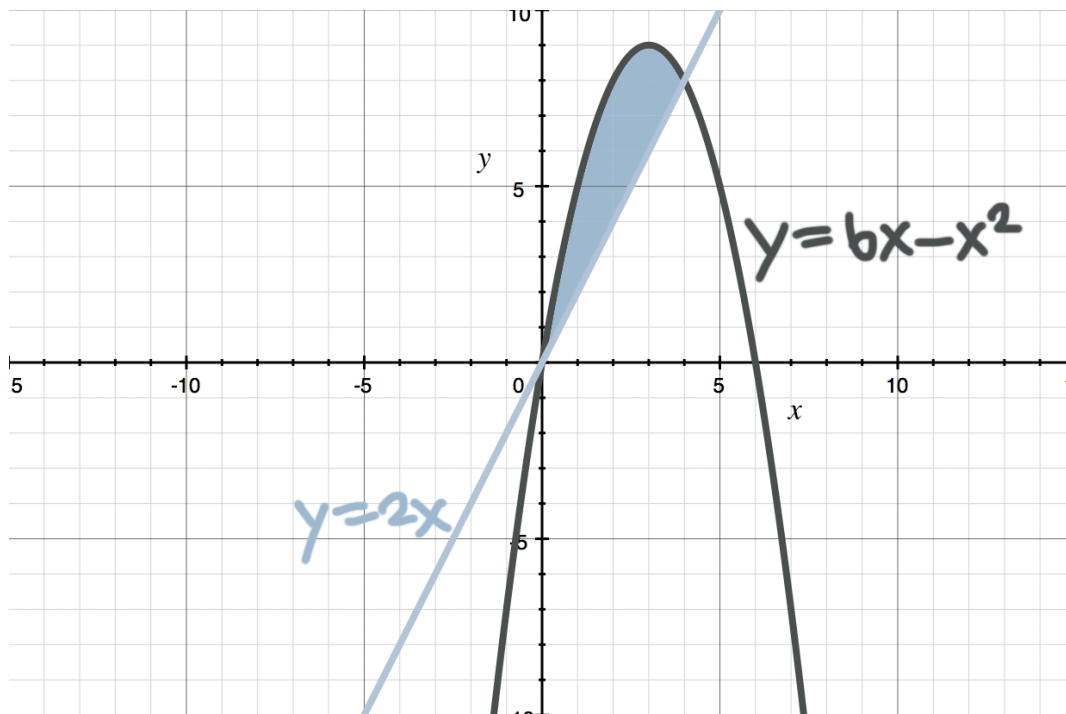
$$4 \cdot \frac{9\pi}{8} = \frac{9\pi}{2}$$

And finally, the area of the shaded region is

$$9\pi - \frac{9\pi}{2} = \frac{9\pi}{2} \approx 14.137$$

38. Correct answer: (C)

Sketch the graph.



Now find the points of intersection.

$$2x = 6x - x^2$$

$$0 = 4x - x^2$$

$$x(4 - x) = 0$$

$$x = 0, 4$$

To find the volume about the y -axis, we use

$$V = 2\pi \int_0^4 x(6x - x^2 - 2x) dx$$

$$V = 2\pi \cdot \frac{64}{3} \approx 134.041$$

39. Correct answer: (B)

Parallel tangents will be when the slopes of f and g are equal. First, find the derivative of the functions.

$$f'(x) = 2e^{3x} \cdot 3 = 6e^{3x}$$

$$g'(x) = 15x^2 - 2$$

Find x where $f'(x) = g'(x)$.

$$6e^{3x} = 15x^2 - 2$$

$$x \approx -0.478$$

40. Correct answer: (C)

We need to take the derivative of the general term with respect to x .

$$f'(x) = \sum_{n=1}^{\infty} \frac{(-1)^n (2n-1)x^{2n-2}}{2n-1} = \sum_{n=1}^{\infty} (-1)^n x^{2n-2}$$

41. Correct answer: (B)

We need to find the second derivative of f and find its x -intercept.

$$f'(x) = 6x^2 + 6x - 2 \cos x$$

$$f''(x) = 12x + 6 + 2 \sin x$$

$$12x + 6 + 2 \sin x = 0$$

$$x \approx -0.430$$

42. Correct answer: (C)

Since $F'(x) = f(x)$, then $F(x) = \int f(x) dx$. Therefore, we get

$$F(6) - F(0) = \int_0^6 f(x) dx$$

From the graph, we can write

$$F(6) - F(0) = \int_0^2 f(x) dx + \int_2^4 f(x) dx + \int_4^6 f(x) dx$$

where $\int_2^4 f(x) dx$ is the area of the rectangle and $\int_4^6 f(x) dx$ is the area of the triangle. Therefore

$$F(6) - F(0) = 4.2 + 2(3) + \frac{1}{2}(2)(3) = 4.2 + 6 + 3 = 13.2$$

43. Correct answer: (B)

The right-hand sum is

$$f(t_1)\Delta t + f(t_2)\Delta t + \dots + f(t_6)\Delta t$$

Since we have six equal subintervals of $[0,12]$, we get

$$2(2.3 + 3.1 + 1.0 + 4.5 + 6.2 + 4.6) = 2(21.7) = 43.4$$

44. Correct answer: (A)

We know that $v(t) = s'(t)$. Since we have linear position, therefore, we get constant velocity, and the correct graph is A.

45. Correct answer: (D)

$$\begin{aligned}\lim_{x \rightarrow a} \frac{x^3 - a^3}{x^2 - a^2} &= \lim_{x \rightarrow a} \frac{(x - a)(x^2 + ax + a^2)}{(x - a)(x + a)} \\ &= \lim_{x \rightarrow a} \frac{x^2 + ax + a^2}{x + a} = \frac{a^2 + a^2 + a^2}{2a} = \frac{3a^2}{2a} = \frac{3}{2}a\end{aligned}$$

CALCULUS AB

SECTION II, Part A Solutions

1. Solution:

a. To find the average value, we use the following formula:

$$\begin{aligned} r_{avg} &= \frac{1}{20-0} \int_0^{20} \frac{1}{10}(h^2 - 6) dh \\ &= \frac{1}{200} \left(\frac{h^3}{3} - 6h \right)_0^{20} = \frac{1}{200} \left(\frac{20^3}{3} - 6(20) \right) = \frac{1}{200} \cdot \frac{7,640}{3} = \frac{191}{15} \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{b. } V &= \pi \int_0^{20} \left(\frac{1}{10}(h^2 - 6) \right)^2 dh \\ &= \frac{\pi}{100} \int_0^{20} h^4 - 12h^2 + 36 dh = \frac{\pi}{100} \left(\frac{h^5}{5} - 4h^3 + 36h \right) \Big|_0^{20} \\ &= \frac{\pi}{100} \left(\frac{20^5}{5} + 4(20^3) + 36(20) \right) = \frac{\pi}{100} (640,000 - 32,000 + 720) \\ &= \frac{60,832\pi}{10} = \frac{30,436\pi}{5} \text{ cm}^3 \end{aligned}$$

$$\text{c. } \frac{r}{h} = \frac{10}{20} = \frac{1}{2}, \text{ so } r = \frac{1}{2}h.$$

The volume of cone is

$$V = \frac{1}{3}\pi r^2 h$$

So we get

$$V = \frac{1}{3}\pi \left(\frac{1}{2}h\right)^2 h = \frac{1}{12}\pi h^3$$

Then

$$\frac{dV}{dt} = \frac{1}{12}\pi \left(3h^2 \frac{dh}{dt}\right)$$

$$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$$

If $h = 5$ and $\frac{dh}{dt} = -\frac{2}{5}$, then

$$\frac{dV}{dt} = \frac{1}{4}\pi(5)^2 \left(-\frac{2}{5}\right) = -\frac{5}{2}\pi \text{ cm}^3/\text{hr}$$

2. Solution:

a. From the graph we see that if $t = 1$, then $y(1) = 2$. Now find $x(1)$.

$$x(1) = x(0) + \int_0^1 x'(t) dt$$

Since at $t = 0$ the particle is at position $(3,1)$, then $x(0) = 3$. We get

$$\begin{aligned} x(1) &= 3 + \int_0^1 x'(t) dt \\ &= 3 + \int_0^1 t^2 + 2 \sin(2t^2 + 1) dt = 3 + 1.996 = 4.996 \end{aligned}$$

Therefore, the position of the particle at $t = 1$ is $(4.996, 2)$.

b. To find the slope of the line tangent to the path of the particle, we need to find $\frac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{y'(t)}{x'(t)}$$

$$\frac{dy}{dx}(1) = \frac{y'(1)}{x'(1)}$$

$$y'(1) = 1$$

$$x'(1) = 1^2 + 2 \sin(2 \cdot 1^2 + 1) = 1.282$$

Then

$$\frac{dy}{dx}(1) = \frac{1}{1.282} = 0.78$$

c. To find the speed, we use the formula $\sqrt{(x'(t))^2 + (y'(t))^2}$.

$$x'(5) = 5^2 + 2 \sin(2 \cdot 5^2 + 1) = 25 + 2 \sin 51 \approx 26.340$$

$$y'(5) = 1.5$$

Then the speed is

$$\sqrt{(26.340)^2 + (1.5)^2} = 26.383$$

d. Now calculate the distance.

$$\int_0^4 \sqrt{(x'(t))^2 + (y'(t))^2} dt = \int_0^2 \sqrt{(x'(t))^2 + (1)^2} dt + \int_2^4 \sqrt{(x'(t))^2 + 0^2} dt$$

$$\approx 4.840 + 18.447 \approx 23.287$$

CALCULUS AB

SECTION II, Part B Solutions

3. Solution:

a. $g(-2) = \int_{-3}^{-2} f(t) dt = \frac{1}{2}(2)(2) = 2$ because the area of a triangle is $\frac{1}{2}bh$.

$$g'(x) = f(x)$$

$$g'(-2) = f(-2) = 1$$

$$g''(x) = f'(x)$$

$$g''(-2) = f'(-2)$$

The slope of f is

$$m = \frac{-2 + 3}{1 - 2} = \frac{1}{-1} = -1$$

$$g''(-2) = -1$$

b. Since $g'(x) = f(x)$ and $f(x) \leq 0$ for $0 \leq x \leq 2$, the function g has neither a relative minimum nor a relative maximum at $x = 1$.

c. To find where g has a relative minimum, we need to find where $g'(x) = 0$ and where g' changes from negative to positive.

$$g'(x) = f(x)$$

Therefore, $g'(x) = 0$ where $f(x) = 0$: $x = -1$, $x = 1$, and $x = 3$.

Since $g' = f$ changes from negative to positive only at $x = 3$, then g has a relative minimum at $x = 3$.

d. The graph of g has a point of inflection at $x = -3$, $x = 0$, $x = 1$, and $x = 2$ because $g'(x) = f(x)$ changes from increasing to decreasing at $x = -3$ and $x = 1$ and from decreasing to increasing at $x = 0$ and $x = 2$.

4. Solution

a. To sketch a slope field, we need to find slope at each given point.

$$\frac{dy}{dx}(1,0) = \frac{-3(1)^3}{0^3}$$

Because this value is undefined, we have the vertical line at $(1,0)$ and the same vertical line for $(-1,0)$.

$$\frac{dy}{dx}(1,1) = \frac{-3(1)^3}{1^3} = -3$$

$$\frac{dy}{dx}(1,2) = \frac{-3(1)^3}{2^3} = -\frac{3}{8}$$

$$\frac{dy}{dx}(0,1) = \frac{-3(0)^3}{1^3} = 0$$

$$\frac{dy}{dx}(0,2) = 0$$

$$\frac{dy}{dx}(0,-1) = 0$$

$$\frac{dy}{dx}(0,-2) = 0$$

$$\frac{dy}{dx}(-1,1) = \frac{-3(-1)^3}{1^3} = 3$$

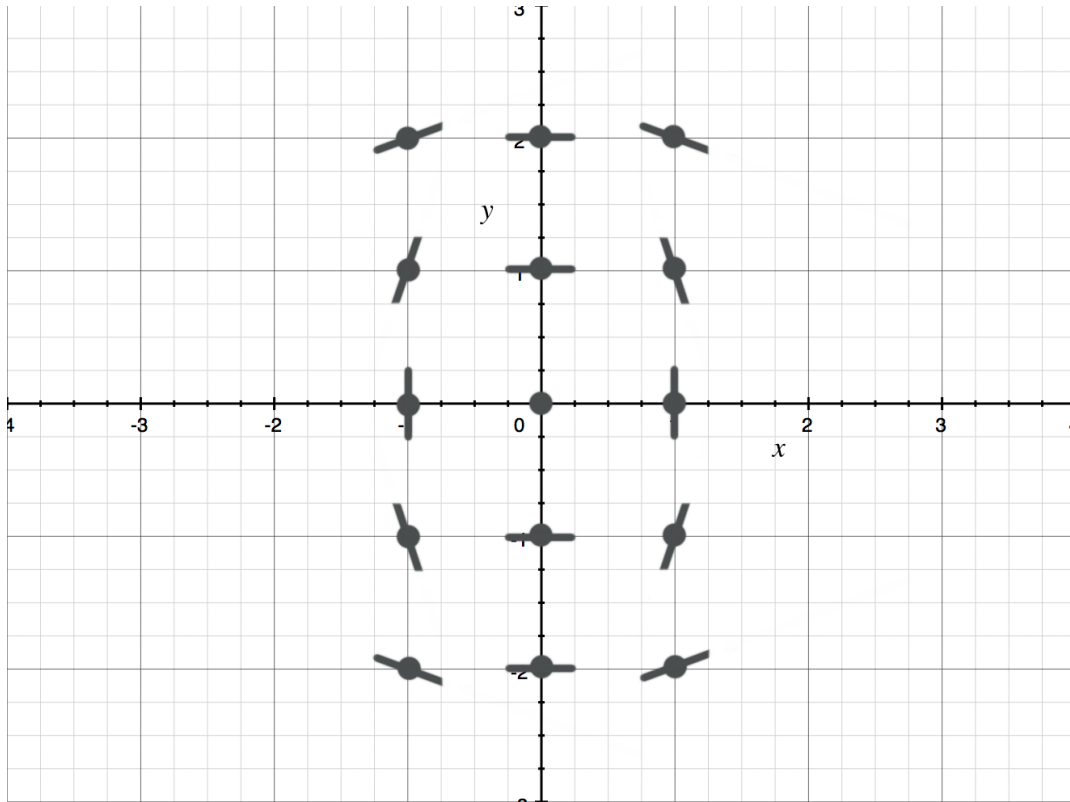
$$\frac{dy}{dx}(-1,2) = \frac{-3(-1)^3}{2^3} = \frac{3}{8}$$

$$\frac{dy}{dx}(-1,-1) = \frac{-3(-1)^3}{(-1)^3} = -3$$

$$\frac{dy}{dx}(-1,-2) = \frac{-3(-1)^3}{(-2)^3} = -\frac{3}{8}$$

$$\frac{dy}{dx}(1, -1) = \frac{-3(1)^3}{(-1)^3} = 3$$

$$\frac{dy}{dx}(1, -2) = \frac{-3(1)^3}{(-2)^3} = \frac{3}{8}$$



b. The slope at (1,2) is

$$\frac{dy}{dx}(1,2) = -\frac{3}{8}$$

Therefore, the line tangent to f at (1,2) is

$$y - 2 = -\frac{3}{8}(x - 1)$$

$$y = -\frac{3}{8}x + \frac{3}{8} + 2$$

$$y = -\frac{3}{8}x + \frac{19}{8}$$

$$f(1.1) = -\frac{3}{8}(1.1) + \frac{19}{8} = 1.963$$

$$c. \frac{dy}{dx} = -\frac{3x^3}{y^3}$$

$$y^3 dy = -3x^3 dx$$

$$\int y^3 dy = \int -3x^3 dx$$

$$\frac{y^4}{4} = -\frac{3}{4}x^4 + C$$

Since $f(1) = 2$, substitute and solve for C .

$$\frac{2^4}{4} = -\frac{3}{4}(1)^4 + C$$

$$4 = -\frac{3}{4} + C$$

$$C = 4 + \frac{3}{4} = \frac{19}{4}$$

Therefore

$$\frac{y^4}{4} = -\frac{3}{4}x^4 + \frac{19}{4}$$

or

$$y^4 = -3x^4 + 19$$

Since the particular solution goes through $(1,2)$, y must be positive, so the particular solution is $y = \sqrt[4]{19 - 3x^4}$.

5. Solution:

$$\text{a. } f'(6) = \frac{f(7) - f(5)}{7 - 5} = \frac{3 - (-2)}{2} = \frac{5}{2} = 2.5$$

$$\text{b. } \int_1^{12} 2f'(x) - 5 \, dx = 2 \int_1^{12} f'(x) \, dx - \int_1^{12} 5 \, dx$$

We know that

$$\int_1^{12} f'(x) \, dx = f(12) - f(1)$$

Therefore, we get

$$\int_1^{12} 2f'(x) - 5 \, dx = 2(f(12) - f(1)) - 5x \Big|_1^{12}$$

$$= 2(5 - 2) - 5(12 - 1) = 2 \cdot 3 - 5 \cdot 11 = 6 - 55 = -49$$

$$\text{c. } \int_1^{12} f(x) \, dx \approx f(1)(2 - 1) + f(2)(5 - 2) + f(5)(7 - 5) + f(7)(8 - 7) + f(8)(12 - 8)$$

$$= 2(1) + 0(3) + (-2)(2) + 3(1) + 7(4)$$

$$= 2 - 4 + 3 + 28 = 29$$

d. To find the equation of the tangent line, we use the formula

$$y = mx + b$$

where $(x, y) = (5, -2)$ and $m = 2$. Substitute and solve for b .

$$-2 = 2(5) + b$$

$$-2 - 10 = b$$

$$b = -12$$

Therefore, we get $y = 2x - 12$. Now we need to find $f(6)$.

$$f(6) = 2(6) - 12 = 0$$

Since $f''(x) < 0$ on $5 \leq x \leq 7$, then 0 is the greatest possible value of $f(6)$.

6. Solution:

a. Use the ratio test.

$$\frac{a_{n+1}}{a_n} = \frac{(-1)^{n+2}(x-1)^{2n+1}}{3^{n+1} \cdot (2n+1)} \cdot \frac{3^n(2n-1)}{(-1)^{n+1}(x-1)^{2n-1}} = -\frac{(x-1)^2(2n-1)}{3(2n+1)}$$

$$\lim_{n \rightarrow \infty} \left| -\frac{(x-1)^2(2n-1)}{3(2n+1)} \right| = \left| \frac{(x-1)^2}{3} \right| \lim_{n \rightarrow \infty} \left| -\frac{2n-1}{2n+1} \right| = \left| \frac{(x-1)^2}{3} \right| \cdot 1$$

Therefore

$$\left| \frac{(x-1)^2}{3} \right| < 1$$

$$|(x-1)^2| < 3$$

$$|x-1| < \sqrt{3}$$

The radius of convergence is $R = \sqrt{3}$.

b. The first four nonzero terms of the Taylor series for f are

$$\frac{x-1}{3} - \frac{(x-1)^3}{27} + \frac{(x-1)^5}{135} - \frac{(x-1)^7}{567}$$

Then the first four nonzero terms of the Taylor series for f' are

$$\frac{1}{3} - \frac{(x-1)^2}{9} + \frac{(x-1)^4}{27} - \frac{(x-1)^6}{81}$$

And the general term is

$$\frac{(-1)^{n+1}(x-1)^{2n-2}}{3^n} \text{ for } n \geq 1$$

c. The first four nonzero terms of the Maclaurin series for $\ln(1+x)$ about $x = 1$ are

$$(x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4}$$

The product of the Maclaurin series for $\ln(1+x)$ and the Taylor series for f is

$$\begin{aligned} & \left((x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \dots \right) \left(\frac{x-1}{3} - \frac{(x-1)^3}{27} + \frac{(x-1)^5}{135} - \frac{(x-1)^7}{567} + \dots \right) \\ &= \frac{(x-1)^2}{3} - \frac{(x-1)^3}{6} + \dots \end{aligned}$$

The third-degree polynomial for $g(x) = \ln(1+x)f(x)$ about $x = 1$ is

$$T_3(x) = \frac{(x-1)^2}{3} - \frac{(x-1)^3}{6}$$